Sketching parabolas with factorisation

Tuesday, 9 October 2018 4:40 PM

By the end of the lesson I would hope that you have an understanding of the following. I would also hope that you can apply the understanding to a number of different questions and question types

- How to factorise using the T-Method and the Null Factor Law
- How to find the x-axis intercepts
- How to find the turning points from the x-axis intercepts
- How to find the y-axis intercept
- How to use the above information to draw a sketch

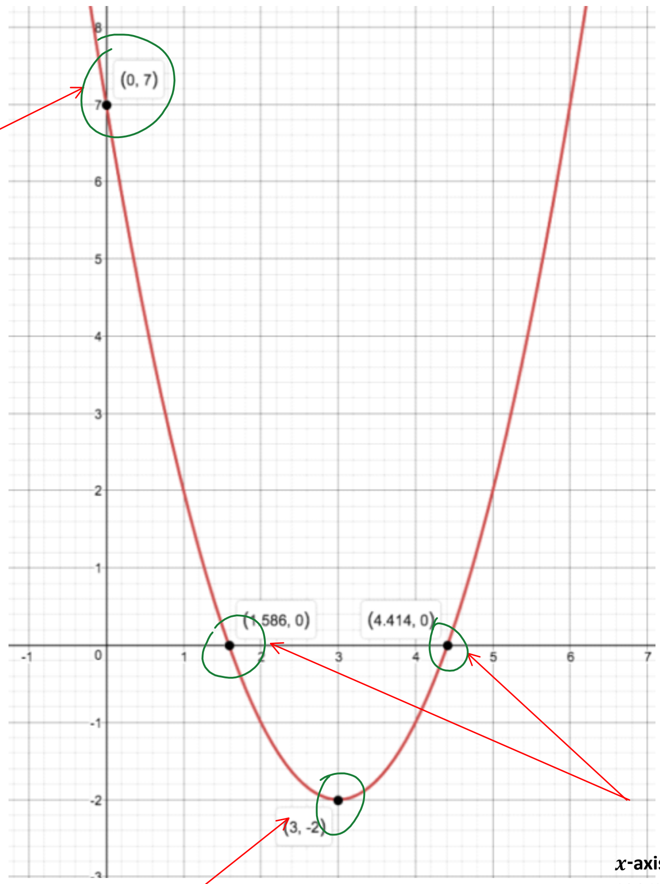
Recap:

We have, in previous lessons, spent a lot of time learning how to factorise and solve quadratics. ✓
We have come to the opinion that, we do all this hard work in order to give us 4 main pieces of information:

Note: The name for the shape of a quadratic is **parabola**.

CTS : Turning pts
T . x-axis ints
QE : $b^2 - 4ac$

y-axis intercept



x-axis intercept(s) also in coordinates!

Turning point (coordinate!)

We are now going to use all that algebra to now sketch some curves.
There are **four** main ways of using the Mathematics to sketch a parabola:

- Using transformations
- Using Factorisation
- Using Completing the Square
- Using the Quadratic Formula

This lesson is going to recap the work for T-Method and factorising using the Null Factor Law and show how we can use them to sketch parabolas.

RECAP: T-Method

The T-Method is one way of factorising a quadratic to place it into **binomial product** form.
Once it's in the **binomial product form** we know we use the **null factor law** to find two solutions.

Example:
Factorise $y = x^2 + x - 6$ 3

$y = x^2 - 2x + 3x - 6$

$$\begin{array}{c|c} \multicolumn{2}{c}{-6} \\ -1 & 6 \\ -2 & 3 \end{array}$$

Factorise $y = x^2 + x - 6$

$y = x^2 - 2x + 3x - 6$

$= x(x-2) + 3(x-2)$

$= (x-2)(x+3)$

	-6
-1	6
-2	3
-3	2
-6	1

Example:
Factorise $y = 2x^2 + 5x - 12$

$y = 2x^2 - 3x + 8x - 12$

$= x(2x-3) + 4(2x-3)$

$= (2x-3)(x+4)$

	-24
-1	24
-2	12
-3	8
-4	6

Once they have been put into **binomial product form** we can solve the equations for $y = 0$ and use the **null factor law** to solve them two find two values of x. These values happen to be the x-axis intercepts.

Example:
Solve $x^2 + x - 6 = 0$

$0 \times 0 = 0$

$\boxed{x} \times 0 = 0$

$0 \times \boxed{x} = 0$

$y = x^2 + x - 6$

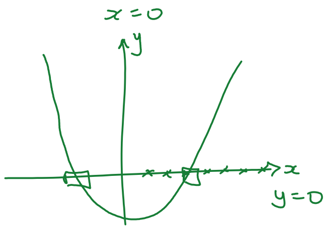
$x^2 + x - 6 = y$

$x^2 + x - 6 = 0$

NFL $(x-2)(x+3) = 0$

$x - 2 = 0$ or $x + 3 = 0$

$x = 2$ $\quad x = -3$



Example:
Solve $2x^2 + 5x - 12 = 0$

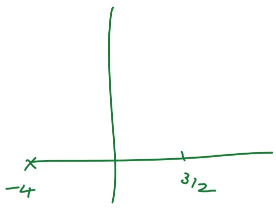
$2x^2 + 5x - 12 = 0$

NFL $(2x-3)(x+4) = 0$

$2x - 3 = 0$ or $x + 4 = 0$

$2x = 3$ $\quad x = -4$

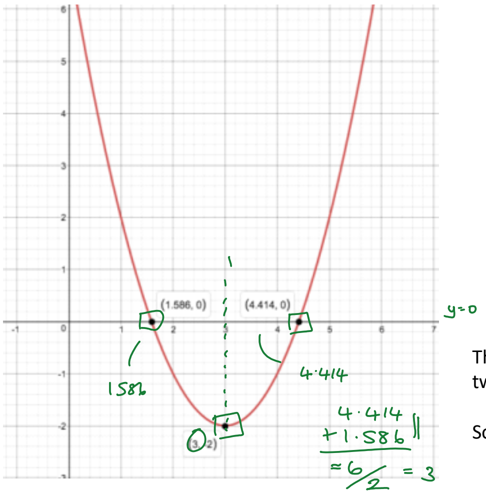
$x = \frac{3}{2}$



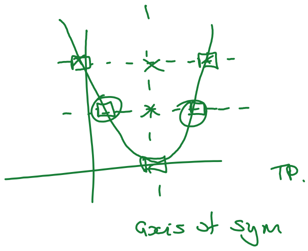
Recap: Finding the turning point from two x-axis intercepts

We have spent a lot of time solving the quadratic equations but not a lot of time drawing them.
We must remember, at all times, that a quadratic equation is **symmetrical**. This **line of symmetry** is actually drawn through the turning point.

$y = x^2 + 3x + 7$



$4.414 + 1.586 = 6/2 = 3$



The great thing we notice that that the x-value of the turning points happens to lie exactly between the two x-values of the intercepts on the x-axis.

So, looking at the example on the left:

$$\boxed{\frac{1.586 + 4.414}{2} = 3}$$

This is an important observation.

Knowing that the x-value of the turning point lies between the two values of the x-axis intercepts, we can find the coordinate of the turning point.

Example:
Find the turning point of: $\boxed{y = x^2 + x - 6}$

$x = 2 \qquad x = -3$

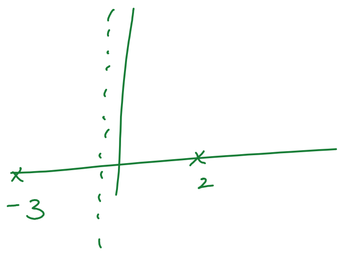
$$\frac{2 + (-3)}{2} = \frac{-1}{2}$$

$$y = \left(-\tfrac{1}{2}\right)^2 + \left(-\tfrac{1}{2}\right) - 6$$

$$= \frac{1}{4} - \frac{1}{2} - 6$$

$$y = -\frac{1}{4} - 6$$

$x = -1/2$



$(-1/2, -6\tfrac{1}{4})$

x

Example:
Find the turning point of $y = 2x^2 + 5x - 12$

$x = \frac{3}{2} \qquad x = -4$

$1.5 - 4$

$$\frac{-2.5}{2} = -1.25$$

$$\frac{\frac{3}{2} + (-4)}{2} \qquad \frac{\frac{3}{2} - 4}{2}$$

T.P. $(-1.25, -15.125)$

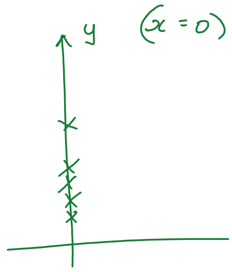
We now have three pieces of information!
We can find the final piece by considering the fact that the y-intercept is found at the point where x is zero.

Example:
Find the y-axis intercept of: $y = x^2 + x - 6$

$x = 0$

$$y = (0)^2 + 0 - 6$$

$$= -6$$



Example:
Find the y-axis intercept of $y = 2x^2 + 5x - 12$

$x = 0 \qquad y = \cancel{2(0)^2} + \cancel{5(0)} - 12$

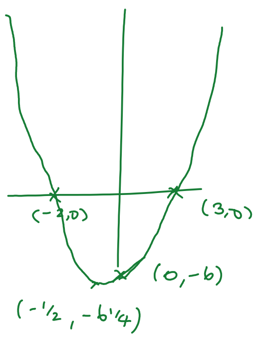
$= -12$

Bringing it home

We now have all the information we need to be able to sketch the graphs we have been considering:

Example:
Sketch the graph of: $y = x^2 + x - 6$

1. TP $(-\frac{1}{2}, -6\frac{1}{4})$
2. X. INT -2
3. X INT $+3$
4. Y. INT -6



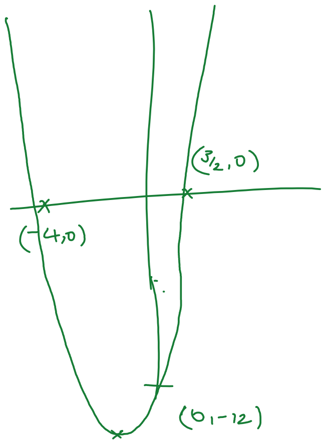
Example:
Sketch the graph of $y = 2x^2 + 5x - 12$

$(-1.25, -15\ 125)$

$3/2$

-4

-12



Tips and Tricks

Remember that Maths is going to try and trick you!
You need to remember that we only use the T-Method when there are three terms which need to be factorised.

When there are two terms this is going to be a massive trick!

Example:
Sketch $y = x^2 + 4x$ 0

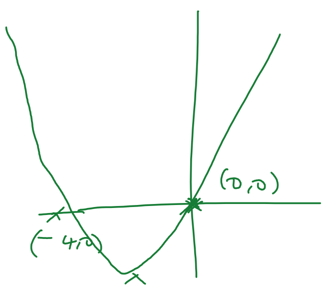
$y = x(x+4)$

$x(x+4) = 0$

$x = 0 \qquad x = -4$

$\frac{0 + (-4)}{2}$

$x = -2$



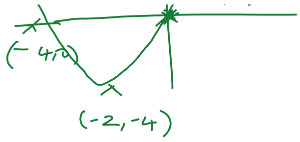
$\overline{2}$

$x = -2$

$y = (-2)^2 + 4(-2)$

$= 4 - 8$

$= -4$



Example:
Sketch $y = x^2 - 16$

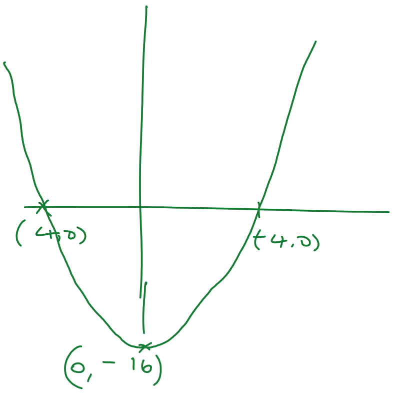
$y = (x+4)(x-4)$

$(x+4)(x-4) = 0$

$x = 4$ or $x = -4$

TP : $x = 0$

$y = -16$



Example:
Sketch $y = x^2 + 4x + 4$

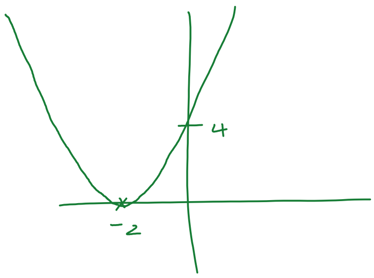
$y = (x+2)^2$

y int

$x = 0$

$y = (0+2)^2$

$= 4$



Example:
Sketch the graph of $y = 9 - x^2$

$y = (3-x)(3+x)$

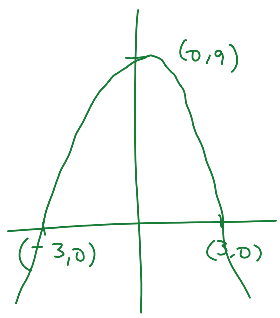
$3 - x = 0$ or $3 + x = 0$

$x = 3$ $\qquad$ $x = -3$

$y = 9 - (0)^2$

$= 9$

$x = 0$



$\boxed{y = ax^2 + bx + c}$