Conditional Probability

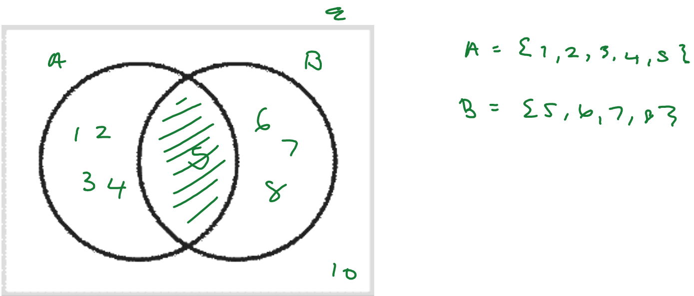
Saturday, 3 November 2018 3:42 pm

By the end of the lesson I would hope that you have an understanding of the following. I would also hope that you can apply the understanding to a number of different questions and question types

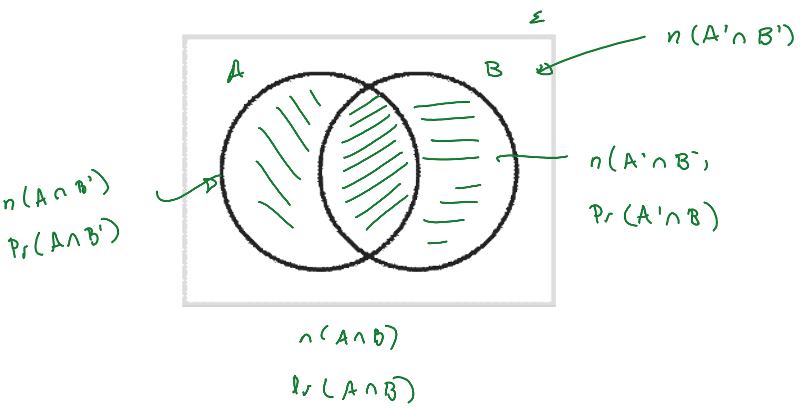
- What it means to have conditional probability
- How to find the conditional probability from a Venn Diagram
- How to find conditional probability using formulae

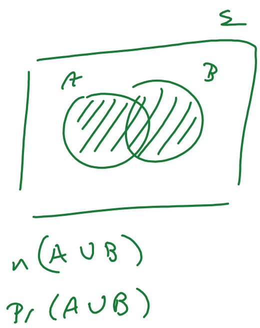
RECAP:

In previous lessons I have already looked at the ideas of using Venn Diagrams to represent events happening. Venn Diagrams are awesome!



Each section of the Venn Diagram stands for something and we can use this to help us find probabilities or numbers of items (depending on what the question is asking from us).





The weather seems to be very interesting to people in the UK and Melbourne!
Who calls their website BOM?

When we say "Hello Google. Is it going to rain today?" .... She might respond:

"There is a 70% chance of rain today."

But this is conditional on lots of things. Examples might be:

- A cold front coming to Mount Eliza
- Rain Clouds forming
- Another weather front coming in and pushing the rain clouds away.

Conditional probability is really important in Maths

Sadly, this is an area of Mathematics which confuses many.
I think I've found the BEST way to explain it!

Firstly, we define conditional probability as:

The probability one thing happens given that something else has already happened.

In Maths we can express this using the following notation

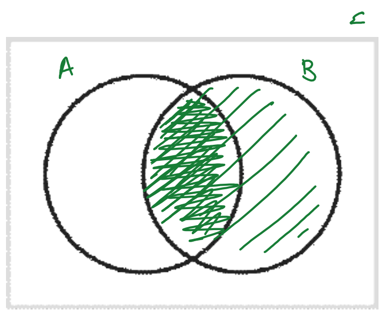
$$\boxed{\Pr(A|B)}$$

Pr (A | B)

A given that B

Let's explain what this means ... using a Venn Diagram:

P(A|B)



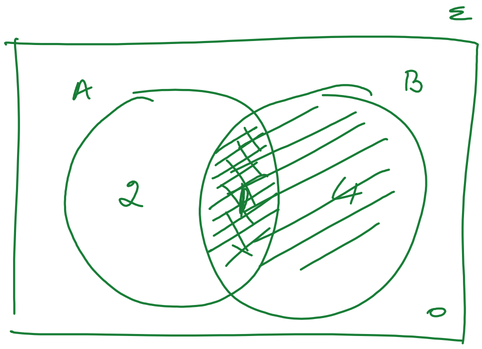
If event B has already happened then we are now contained within the event B. This makes sense!
We now look at the overlap to see what chance we now have of event A happening.

As we are looking for a probability we are looking for the number of ways A can now happen divided by the number of ways B can happen.

$n(A|B) = \frac{1}{5}$

$n(B) = 5$

$n(A \cap B) = 1$



$Pr(A|B) = \frac{1}{7} \div \frac{5}{7} = \frac{1}{7} \times \frac{7}{5} = \frac{1}{5}$

$Pr(B) = \frac{5}{7}$

$Pr(A \cap B) = \frac{1}{7}$

This can be more formally stated in an formula:

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$

This can also be written the other way:

$$\Pr(B|A) = \frac{\Pr(B \cap A)}{\Pr(A)}$$

$Pr(B|A) = Pr(B \cap A)$

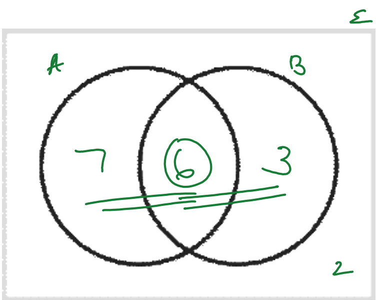
$Pr(A)$

Note: $\mathbf{Pr}(\boldsymbol{A} \cap \boldsymbol{B}) = \mathbf{Pr}(\boldsymbol{B} \cap \boldsymbol{A})$

Examples

Consider the following Venn Diagram displaying the number of elements belonging to two events, A and B

$Pr(B|A) = \frac{6}{13}$



$\frac{6}{9}$

$Pr(B) = \frac{9}{18}$

Use the Venn Diagram to find:

A. $\Pr(A)$ $= \frac{n(A)}{n(\varepsilon)} = \frac{13}{18}$

B. $\Pr(A \cap B)$ $= \frac{n(A \cap B)}{n(\varepsilon)} = \frac{6}{18} = \frac{1}{3}$

C. $\Pr(A|B)$ $= \frac{Pr(A \cap B)}{Pr(B)} = \frac{6}{18} \div \frac{9}{18} = \frac{6}{\cancel{18}} \times \frac{\cancel{18}}{9} = \frac{6}{9} = \frac{2}{3}$

D. $\Pr(B|A)$ $= \frac{Pr(B \cap A)}{Pr(A)} = \frac{6}{18} \div \frac{13}{18} = \frac{6}{\cancel{18}} \times \frac{\cancel{18}}{13} = \frac{6}{13}$

Example taken from the Cambridge Essentials Textbook

This is an awesome resource which I am using to teach my students

From a group of 15 hockey players at a game of hockey, 13 played on the field, 7 sat on the bench and 5 both played and sat on the bench.

A hockey player is chosen at random from the team.

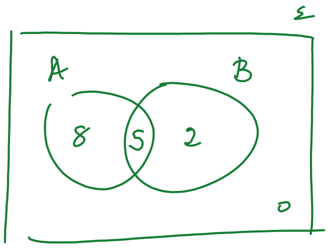
Let A be the event 'the person played on the field' and let B be the event 'the person sat on the bench'.

a Represent the information in a two-way table.

b Find the probability that the person only sat on the bench.

c Find the probability that the person sat on the bench given that they played on the field.

d Find the probability that the person played on the field given that they sat on the bench.



	A	A'		
B	$\boxed{5}$	2	$\boxed{7}$	$n(B)$
B'	8	0	8	
	$\boxed{13}$	2	15	$n(\varepsilon)$

b) $Pr(B) = \frac{7}{15}$

c) $Pr(B \mid A) = \frac{Pr(A \cap B)}{Pr(A)} = \frac{5}{13}$

d) $Pr(A \mid B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{5}{7}$