Binomial Products

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By the end of the lesson I am hoping you will be able to understand and apply the knowledge to:

- Know what a binomial expansion is
- Be able to use the ideas behind the binomial expansion to multiply out a set of brackets
- Simplify, where possible, an binomial expansion into simplest form.

RECAP:

In previous lessons we have spent time learning :

- What an irrational number is
- What a surd is
- Adding surds
- Subtracting surds
- Simplifying surds
- Multiplying surds
- Dividing surds
- Squaring surds

This lesson is going to continue to build on the idea of surds and how we can multiply using the **distributive law** a couple of **binomial products**.

What is a binomial product?
A binomial product is (generally) when we have two expressions multiplied together.
This is a topic area we will see a lot of in the coming months and years.
Hence, $(x+3)(x-4)$ can be idenitified as a binomial product.

$(x+3)(x-4)$

Later, in Year 10, we will be doing a LOT more work on Binomial Expansions as this is a key area of Mathematics in Year 11 and 12. We will look at something called the Binomial Expansion.

Expanding Binomial products (without surds!)

When we multiply two binomial products together we are actually following a bit of a pattern.
Normally we teach this the hard way!
I want to go back to Year 7 Maths and make this slightly simpler.

How would I multiply 23 and 35 together?
There are two ways! The **easy way** and the **hard way**.
From junior school you've been shown the **HARD** way!

$23 \times 35 =$

$$\begin{array}{r} 23 \\ \times\ 35 \\ \hline 115 \\ +\ 690 \\ \hline 805 \\ \hline \end{array}$$

The easy way can actually be used to show how to expand expressions of the form $(x+3)(x-2)$.
In the same way as 23 and 35 can be thought of as 20 + 3 and 30 + 5, we can use the same ideas:

23   20 + 3
35   30 + 5

715   700 10 5
23   20 3

×	20	3
30	600	90
5	100	15

600   90
100   15
700 + 105 = 805

I was taught a different way using something called FOIL.
I never called it foil ...
This is how I was taught to do $(x + 3)(x - 2)$

$\times$	x	$+3$
x	x^2	$+3x$
-2	$-2x$	-6

$\cancel{x^2} + \cancel{3x} \cancel{-2x} - 6 = x^2 + x - 6$

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$(x+3)(x-2)$

$= x^2 - 2x + 3x - 6$

$= x^2 + x - 6$

What does this have to do with Surds?
Well, what we can do with numbers, we can do with Surds!
As surds are numbers too ...

Multiplying Binomial Products containing Surds

Let's use both ways shown above to find $(4 + \sqrt{2})(\sqrt{2} - 3)$

$\times$	4	$+\sqrt{2}$
$+\sqrt{2}$	$4\sqrt{2}$	2
-3	-12	$-3\sqrt{2}$

$\boxed{4\sqrt{2}} + 2 - 12 \boxed{-3\sqrt{2}}$

$\sqrt{2} - 10$

$(4 + \sqrt{2})(\sqrt{2} - 3)$

$\boxed{4\sqrt{2}} - 12 + 2 \boxed{-3\sqrt{2}}$

$= \sqrt{2} - 10$

How about this example: $(2\sqrt{6} - 1)(3\sqrt{6} + 5)$

$(2\sqrt{6} - 1)(3\sqrt{6} + 5)$

$6 \times 6 + 10\sqrt{6} - 3\sqrt{6} - 5$

$36 + 7\sqrt{6} - 5$

$7\sqrt{6} + 31$

Throwing a curve ball

It always amazes me the number of people who get the following question types wrong!
If you can get this sorted now ... your work in Year 10, 11 and 12 will be awesome :)

We know that $(ab)^2$ is equal to a^2b^2.
We know that when terms inside the brackets are multiplied together we just square each of the terms.
Easy right!

Yup .... But people then use this rule incorrectly when given questions like $(a + b)^2$.

THESE TWO ARE not THE SAME.
THEY DON'T EVEN LOOK THE SAME!!!

$(ab)^2 \neq (a + b)^2$

$(ab)^2 \neq (a+b)^2$

$a^2b^2 \neq a^2 + 2ab + b^2$

We need to be aware that $(a+b)^2$ is the same as $(a+b)(a+b)$ which is a binomial product and, as such, needs to be expanded using FOIL or another method.

For example:

$(x+3)^2 = (x+3)(x+3)$

$= x^2 + 3x + 3x + 9$

$= x^2 + 6x + 9$

We can use the same ideas then with surds!

Expand and simplify the following terms

$\left(3-\sqrt{2}\right)^2 = (3-\sqrt{2})(3-\sqrt{2})$

$= 9 - 3\sqrt{2} - 3\sqrt{2} + 2$

$= 7 - 6\sqrt{2}$

$\left(2\sqrt{3}+3\sqrt{2}\right)^2$ $(2\sqrt{3}+3\sqrt{2})(2\sqrt{3}+3\sqrt{2})$

$4\times 3 + 6\sqrt{6} + 6\sqrt{6} + 9\times 2$

$= 12 + 12\sqrt{6} + 18$

$= 12\sqrt{6} + 30$

Work to be completed this lesson

The work I am asking you to complete at the end of teaching is:

Lesson 4	**3D Binomial products (10A)** Let's start: *Show the missing steps* Key ideas Example 9, 10, 11	Exercise 3D Page 185	— 4-7(½) 8-10(½) 12(½), 13 14